You are welcome to ask for hints and help in the classroom, on Slack or email (sparre@uni-potsdam.de) at any time.

Good luck with the tasks!

The dark matter halo of a galaxy from Illustris TNG

The aim of this exercise is to analyse the dark matter (DM) halo of a galaxy from the Illustris TNG-50-Dark simulation (`https://www.tng-project.org/`). This is a simulation without gas and stars, so it only includes dark matter. We will analyse *DM particles*, which represent *collisionless dark matter* following the ΛCDM cosmology. The target is a galaxy at redshift 0 with a mass of around 10^{12} $M_\odot$. This is comparable to the mass of the Milky Way.

Reading and plotting the data

The DM particles in the halo is included in the file,
`GalaxyFromIllustrisTNG50Dark_DM_Subhalo852966.txt`.

The four columns are:

```
x coordinate of DM particle: in units of kpc
y coordinate of DM particle: in units of kpc
z coordinate of DM particle: in units of kpc
M: mass in units of Msun
```

The coordinates are chosen such that the galaxy centre is in (0,0,0). The data can be read in python using the `numpy.loadtxt` function:

```
import numpy
Array = numpy.loadtxt(
"GalaxyFromIllustrisTNG50Dark\_DM\_Subhalo852966.txt"
)
Pos = Array[:,0:3]
Mass = Array[:,3]
```

Task 1 Plot DM mass projections of the galaxy halo (in units of $M_\odot \mathrm{kpc}^{-2}$). Do (x, y), (x, z) and (y, z) projections. Choose a box size of $\pm$300 kpc. Hint: Use plt.hist2d to create the projections.

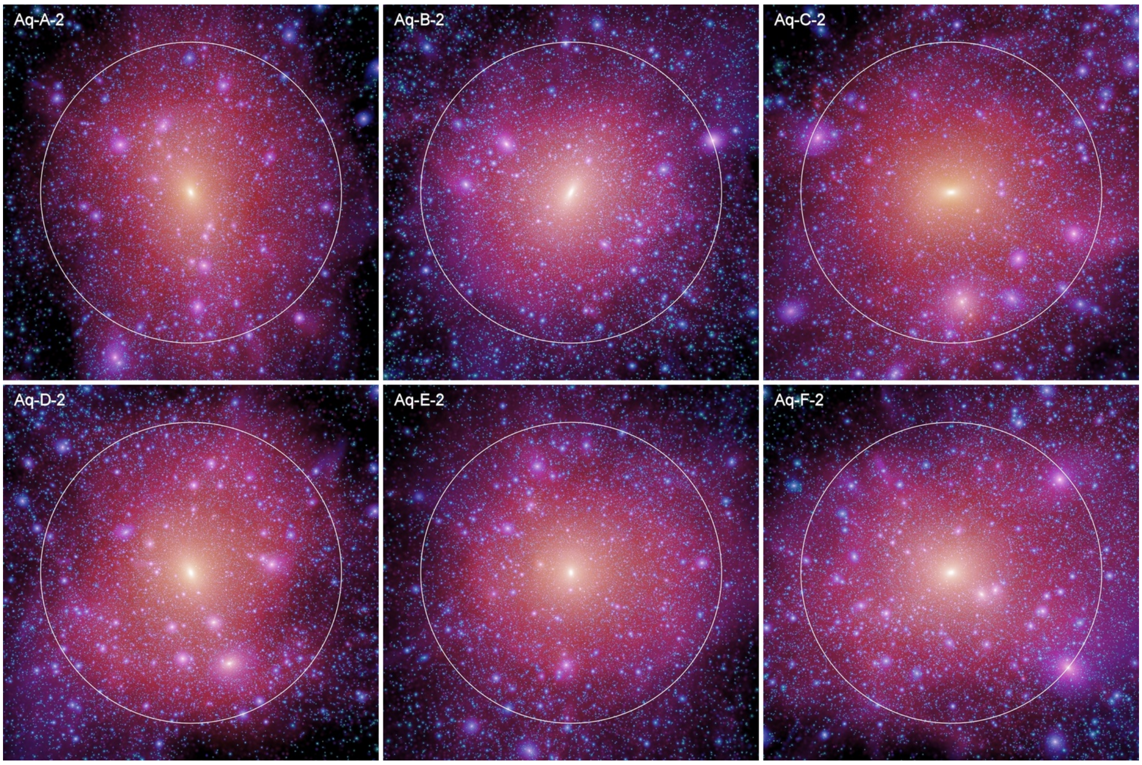


Figure 1: Six different simulated dark matter haloes from the Aquarius simulation project. For all simulations, the main halo is roughly spherical with a density peak in the centre. There is a rich substructure consisting of many small satellite galaxies. For more details, see Springel et al. 2008, `https://arxiv.org/abs/0809.0898`.

Satellite / neighbour galaxies

Simulated dark matter halos have a rich substructure and contain many *satellite galaxies*. See for example Fig. 1, which shows six simulated halos from the Aquarius simulation project (Springel et al. 2008, `https://arxiv.org/abs/0809.0898`).

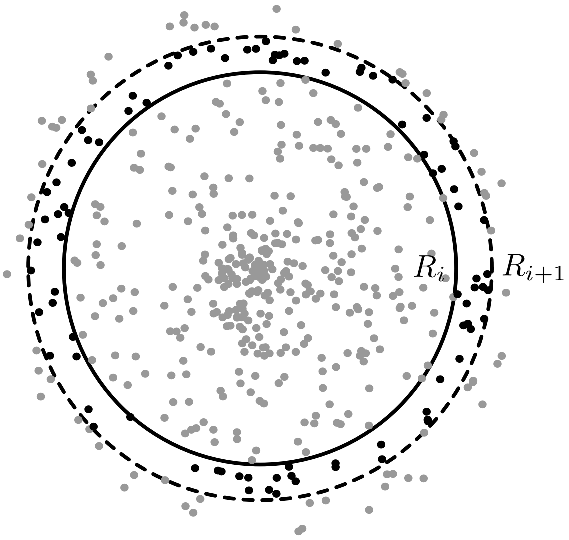
In the above galaxy from Illustris TNG-50-Dark, the 10 most massive galaxies near our selected galaxy have the following x, y, z coordinates:

```
0. 0. 0.
 28.9307  147.77176  91.3073
159.32645  -27.599352 -80.43161
-29.184427 266.3879   -95.67256
-10.797834 148.63385  -17.152555
-44.74542   58.259293 -40.10336
 -82.00299   89.14701 -168.42316
39.252796 20.820791 19.300589
 162.94783  -146.49446    32.405037
170.69516  -58.414986 -39.244144
```

Task 2 Mark the locations of these galaxies in the projections from Task 1. Use circles or "x" symbols as markers.

The dark matter density profile

We will now compute the mass density profile (ρ) as a function of the radius, $R \equiv \sqrt{x^2 + y^2 + z^2}$. To do this, we divide the halo into spherical shells. Shell number i has an inner boundary of R_i and outer boundary R_{i+1}. Fig. 2 illustrates how the average density of dark matter particles can be calculated in shell number i:



Inner shell boundary at R_i: solid circle

Outer shell boundary at R_{i+1}: dashed circle

$$\text{Volume of shell} = \frac{4}{3}\pi[R_{i+1}^3 - R_i^3]$$

$$\text{Mass in shell} = \sum_{j \in \text{black points}} M_j$$

Density of shell = Mass in shell / Volume of shell

Figure 2: A visualisation of how the shell averaged dark matter density is calculated.

We choose the shell boundaries R_i such that they are evenly distributed in $\log R$. In Python, the boundaries of the 40 shells (in units of kpc) can be calculated as:

```
import numpy
logR = numpy.linspace(-1.0,2.5,41)
R = 10.0**logR
```

The first shell is bounded by the radii, `R[0]` and `R[1]` (using Python-notation). Using mathematical notation, this shell has $i = 0$, and it is bounded by R_0 and R_1.

An array with the inner boundaries of the 40 bins is then `R[:-1]` (R_i) and the outer boundaries are contained in the array `R[1:]` (R_{i+1}). The volume per shell, $\frac{4}{3}\pi(R_{i+1}^3 - R_i^3)$, can be calculated in Python as,

```
VolumeOfShell = 4.0/3.0*numpy.pi*((R[1:])**3-(R[:-1])**3)
```

and the geometrical midpoint of each shell, $\frac{1}{2}(R_{i+1} + R_i)$, becomes

```
MidpointShell = 0.5*(R[1:]+R[:-1])
```

Task 3 Calculate the DM mass in each of the spherical shells.

Hint to the above exercise: Calculate the mass in each shell by looping over all bins, and then sum the mass from the DM particles residing in each shell. A fast method is to use the `numpy.digitize` function to calculate the bin of each DM particle, and `numpy.bincount` to calculate the number of particles in each bin. Note, that these functions use a convention, where the first bin is an underflow bin: it counts the number of particles at a radius smaller than `R[0]`. The last bin is an overflow bin with particles with radius larger than `R[-1]`. So these bins have to be removed from the output of `numpy.bincount` to make it compatible with our bin definition (as used by e.g. `MidpointShell`).

Task 4 Calculate ρ as a function of R, and plot $\log \rho$ as a function of $\log R$.

An estimate of radius and mass of a cosmological halo (M_{200} and R_{200})

A commonly used measure of a halo's radius is R_{200}, which is the radius inside which the mean density is 200 times the critical density of the universe. M_{200} is the total mass inside R_{200}.

Task 5 Find the critical density, $\Omega_{\text{crit,0}}$, at redshift 0 in a Planck15 cosmology, for example by using the `astropy` package.

Task 6 Calculate the total mass (M_{tot}) inside a given radius (R) for the above dark matter halo.

Task 7 Solve the equation $200\Omega_{\text{crit,0}} = M_{\text{tot}}/(\frac{4}{3}\pi R^3)$ for the above halo (numerically solve for R). Determine M_{200} and R_{200}.

Task 8 Add a circle indicating R_{200} to the figure from Task 2.